

PHYSICS
Paper – I

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Answers must be written in **ENGLISH** only.

Useful Constants :

Electron charge (e)	= 1.602×10^{-19} C
Electron rest mass (m_e)	= 9.109×10^{-31} kg
Proton mass (m_p)	= 1.672×10^{-27} kg
Vacuum permittivity (ϵ_0)	= 8.854×10^{-12} farad/m
Vacuum permeability (μ_0)	= $4\pi \times 10^{-7}$ T m/A
Velocity of light in free space (c)	= 3×10^8 m/s
Boltzmann constant (k)	= 1.380×10^{-23} J/K
Electronvolt (eV)	= 1.602×10^{-19} J
Planck constant (h)	= 6.626×10^{-34} J s
Stefan constant (σ)	= 5.67×10^{-8} W m ⁻² K ⁻⁴
Avogadro number (N)	= 6.022×10^{26} kmol ⁻¹
Gas constant (R)	= 8.31×10^3 J kmol ⁻¹ K ⁻¹
exp (1)	= 2.718

SECTION A

- Q1.** (a) Define generalized coordinates. How many generalized coordinates are required to describe the dynamics of 4 + 4 = 8
- (i) a free rigid body in 3 dimension ?
- (ii) a solid cylinder rolling in an inclined plane without slipping ?
- (b) Write down the expression for kinetic energy of a relativistic particle with rest mass m_0 , moving with a speed v . 5 + 3 = 8
- (i) Show that the kinetic energy reduces to $\frac{1}{2} m_0 v^2$ in the non-relativistic limit.
- (ii) Find the first relativistic correction to the non-relativistic kinetic energy.
- (c) The Lagrangian of a system is given by

$$L(\vec{r}, \vec{v}) = \frac{1}{2} m (\vec{v} + \vec{a})^2 + \vec{b} \cdot \vec{v} - \vec{c} \cdot \vec{r}$$
 where, \vec{a} , \vec{b} and \vec{c} are constant vectors. Construct the Hamiltonian of the system and find the canonical equations of motion. 8
- (d) Explain the phenomenon of double-refraction in calcite crystal with special reference to Ordinary Ray (OR), Extraordinary Ray (ER) and uniaxial negative crystal. 8
- (e) Draw the energy level diagram of He-Ne laser.
 The metastable state of ruby laser is at 1.786 eV. Calculate the wavelength of the light emitted. 4+4=8

- Q2.** (a) Define Coriolis force. A particle is dropped from a height h vertically at the northern hemisphere at latitude θ . Find its deflection from the vertical line due to Coriolis force. 15
- (b) The moment of inertia tensor of a cube of mass M and side a is given by the matrix

$$I = \frac{Ma^2}{12} \begin{bmatrix} 8 & -3 & -3 \\ -3 & 8 & -3 \\ -3 & -3 & 8 \end{bmatrix}.$$

Calculate the principal moments of inertia of the cube.

10

- (c) (i) Distinguish between interference fringe pattern and diffraction fringe pattern.

5

- (ii) In an experiment using Newton's rings, the diameter of 5th and 15th rings were found to be 33.6×10^{-4} m and 59×10^{-4} m respectively. Calculate the radius of curvature of the plano-convex lens if the source of light is sodium light ($\lambda = 589.0$ nm).

Mention any other applications of Newton's ring experiment.

8 + 2 = 10

- Q3.** (a) Find the equation of orbit for a particle of mass m , moving in the influence of central force $F(\mathbf{r})$ in terms of $u \left(\equiv \frac{1}{r} \right)$.

15

- (b) Consider two inertial frames S and S' . S is at rest and S' is moving along $x - x'$ direction with constant speed v .

10 + 5 = 15

- (i) Find how different components of momentum four-vector in S and S' frames are related.

- (ii) Show that $p^\mu p_\mu$ is Lorentz invariant.

- (c) What are the special features of step-index fibre ?

The step-index fibre has core glass and cladding glass with refractive indices 1.68 and 1.5 respectively. Calculate the numerical aperture. Find the critical angle and acceptance angle for entrance of light if the fibre is placed in air.

3 + 7 = 10

- Q4.** (a) (i) What are the similarities and dissimilarities between a zone plate and a convex lens ?

5

- (ii) A monochromatic light of wavelength 5860×10^{-8} cm is incident normally on a 2 cm wide grating. The first order spectrum is produced at an angle of 20° with respect to the normal. Determine the total number of lines on the grating.

How does the resolving power of grating depend on the number of rulings on the grating ?

7 + 3 = 10

- (b) Construct the Lagrangian of a simple pendulum whose string is replaced by a spring with spring constant k and rest length l_0 oscillating in a vertical $x - y$ plane. Find the equations of motion. 10

- (c) Consider a damped harmonic oscillator given by the equation
$$\ddot{x} + 2\mu \dot{x} + \omega_0^2 x = 0.$$

Obtain the displacements for

- (i) overdamped,
- (ii) underdamped and,
- (iii) critically damped motion. 15

SECTION B

Q5. (a) State Biot–Savart law and write its mathematical form. In case of distributed current sources, write this expression for line current, surface current and volume current. 8

(b) Obtain an expression for Poisson's and Laplace's equations in electrostatics. 8

(c) For a vector potential \vec{A} , $\vec{\nabla} \cdot \vec{A} = -\frac{\mu_0}{4\pi} \cdot \frac{Q}{r^2}$ where Q is a constant of appropriate dimension. Calculate the corresponding scalar potential $\phi(\vec{r}, t)$, that make \vec{A} and ϕ to satisfy Lorentz gauge. 8

(d) A parallel plate capacitor with circular plate of radius ($R = 5$ cm) is being charged. Find out the displacement current.

Given : $\epsilon_0 = 8.9 \times 10^{-12} \text{ C}^2/\text{Nm}^2$, $\frac{dE}{dt} = 10^{12} \text{ V/m}$. 8

(e) Find out the boiling temperature of water at the top of Mount Everest. 8

Given : Pressure at the top of Everest is 0.36 atm.

The density of water vapour at 100°C is 0.598 kg/m^3 .

The latent heat is $2.257 \times 10^3 \text{ J/g}$.

Q6. (a) Briefly explain electronic, ionic and orientational polarisation.

The polarisability of ammonia molecule is found to be $2.42 \times 10^{-39} \text{ C}^2 \text{ m/N}$ and $1.74 \times 10^{-39} \text{ C}^2 \text{ m/N}$ at 309 K and 448 K respectively. Calculate the orientation polarisability for each temperature. 9 + 6 = 15

- (b) An AC circuit consists of inductance of self-induction 'L', a capacitor of capacitance 'C' and a resistor of resistance 'R' in series. Calculate the impedance 'Z' of the circuit.

Obtain resonance condition and define quality factor.

9 + 6 = 15

- (c) Current 'I' is passing through an infinite solenoid of radius R, with n turns per unit length. Find out the vector potential (i) inside the solenoid (ii) outside the solenoid.

10

- Q7.** (a) Distinguish between self-inductance and mutual inductance.

Calculate the self-inductance of a solenoid of length 'l', area of cross-section 'A' having N turns.

5 + 5 = 10

- (b) Explain the cause of Rayleigh scattering. How does the amount of scattering depend on the wavelength of light ?

How does Rayleigh scattering explain :

15

(i) the blue colour of sky at day time ?

(ii) the red colour at dusk ?

- (c) Deduce the thermodynamic relation :

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$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Using the expression establish Clausius-Clapeyron latent heat equation

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

- Q8.** (a) (i) Prove that for perfect gas the specific heat at constant pressure C_p is always greater than the specific heat at constant volume by a constant value R.

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- (ii) Consider for hydrogen, the density at NTP is 0.0899 gm/lit, molecular weight 2.016 gm and specific heat at constant pressure 6.85 cal/gm. Calculate the specific heat of hydrogen gas at constant volume.

Given : $J = 4.18 \times 10^{-7}$ erg.

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- (b) The volume of a mole of liquid He^4 is $27 \times 10^{-6} \text{ m}^3$ and the mass of a He^4 atom is $6.65 \times 10^{-27} \text{ kg}$. Assuming that liquid He^4 is an ideal Bose gas, calculate 10
- (i) the concentration of boson in this volume.
 - (ii) Bose temperature.
- (c) Define electromagnetic field strength tensor $F_{\mu\nu}$. Express it in terms of components of electric and magnetic fields. 15

