Indian Forest Service (Main) Examination, 2024

STATISTICS

Paper – II

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) A subgroup of 5 items each are taken from a manufacturing process at a regular interval. A certain quality characteristic is measured and $\bar{X}$ and R values are computed. After 25 subgroups, it is found that $\Sigma \bar{X} = 357{\cdot}50$ and $\Sigma R = 8{\cdot}80$. If the specification limits are $14{\cdot}40 \pm 0{\cdot}40$ and if the process is in statistical control, what conclusion can you draw about the ability of the process to produce items within specifications. (Given that, for subgroup of 5 items, $d_2 = 2{\cdot}326$, $c_2 = 0{\cdot}8407$, $d_3 = 0$ and $d_4 = 2{\cdot}11$) *8*

(b) Give the relationship between survival function 'S(t)', probability density function 'f(t)', and hazard function 'h(t)'.

Also find : (i) survival function and hazard function for a density function $f(t) = e^{-t}$, $t \geq 0$ and (ii) probability density function and hazard function for the survival function $S(t) = \exp(-t^r)$. *8*

(c) For a homogeneous Markov chain, define the probability of the first return to state i in n steps. Compute the same for a Markov chain with state space {1, 2, 3} and the following transition probability matrix, when i = 2 and n = 3.

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} \end{bmatrix}$$

In addition, if the initial probability $P(X_0 = 1) = \frac{1}{2}$, $P(X_0 = 2) = \frac{1}{4}$ and $P(X_0 = 3) = \frac{1}{4}$, compute the probability $P(X_1 = 1 \mid X_0 \neq 2)$. *8*

(d) Get an estimate of the integral $\int_0^4 \frac{e^{-x}}{x^2 + e^{-4x}} dx$ through Monte Carlo simulation, using the following random numbers : *8*

0·463, 0·802, 0·607, 0·455, 0·37

0·839, 0·401, 0·029, 0·843, 0·811

(e) Solve the following linear programming problem graphically.

Maximize $z = 3x_1 + 2x_2 + 2x_3$

subject to $2x_1 - 3x_2 \leq 6$

$10x_1 + 2x_2 + 3x_3 \leq 40$

$3x_1 + x_3 = 10$

$x_1 \geq 0$, $x_2 \geq 0$ and x_3 is unrestricted.

Obtain the optimal values of x_1, x_2 and x_3. What is the optimal z ? 8

Q2. (a) Suppose that 50 units are placed on life test (without replacement) and the test is to be truncated after 10 of them have failed. It was further supposed that the first 10 failure times are 65, 110, 380, 420, 505, 580, 650, 840, 910 and 950 hours. Estimate the mean life of the component and its failure rate. Also calculate a 90% confidence interval for mean time 'μ'. (Assume that the exponential model holds) 15

Given that :

v	$\chi^2_{0.05}$	$\chi^2_{0.95}$
20	31·41	10·851
21	32·671	11·591

(b) The following table gives the number of missing rivets at aircraft final inspection :

Aircraft Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of missing rivets	8	16	14	19	11	15	8	11	21	12	23	16	9

Aircraft Number	14	15	16	17	18	19	20	21	22	23	24	25
Number of missing rivets	25	15	9	9	14	11	9	10	22	7	28	9

Find $\overline{c}$, compute trial control charts and plot control chart for c. What value of c′ would you suggest for the subsequent period ? 15

(c) Define Reliability. Give its basic elements. Since 'design' is the main area of achieving reliability of a product, give the factors that are to be considered for achieving a reliable design. *10*

Q3. (a) Solve the following linear programming problem using two-phase method : *15*

Maximize $z = 2x_1 + x_2$

subject to $x_1 + 2x_2 \geq 2$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

(b) Tho cost of a new machine is ₹ 5,000. The running cost of n^{th} year is given by $R_n = 500\ (n - 1)$, n = 1, 2 … . Assuming an interest rate of 5% per year, after how many years will it be economical to replace the machinery with a new one ? *15*

(c) The probability distribution of monthly sales of a certain item is as follows :

Monthly sales :	0	1	2	3	4	5	6
Probability :	0·04	0·06	0·25	0·35	0·15	0·09	0·06

The cost of carrying inventory is ₹ 20 per unit per month. The current policy is to maintain a stock of 4 units at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the attributed range of cost of a shortage of one item for one unit of time. *10*

Q4. (a) The following data is on defective electronic machine parts from a standard brand of manufacturer. Sample sizes and number of defectives in each sample are given below. Using suitable control limits, test whether the process is statistically controlled or not. *15*

Sample Size	115	220	210	220	220	255	440
Number of defectives	15	18	23	22	18	15	44

Sample Size	365	255	300	280	330
Number of defectives	47	13	33	42	46

(b) Two companies A and B manufacture similar products and they are competing for an increased market share. The payoff matrix shown in the following table, describes the increase in market share of company A and decrease in market share of company B.

Company B → / Company A ↓	Give Coupons	Decrease Price	Maintain Present Strategy	Increase Advertising
Give Coupons	2	– 2	4	1
Decrease Price	6	1	12	3
Maintain Present Strategy	– 3	2	0	6
Increase Advertising	2	– 3	7	1

Determine the optimal strategies for both the manufacturers and the value of the game. *15*

(c) Mention the names of different sets of tables under Dodge-Romig system. Explain the CSP-1 as developed by Dodge. *10*

SECTION B

Q5. (a) Define Crude Death Rate. Write merits and demerits of Crude Death Rate. Is Crude Death Rate an accurate measure of the mortality of population of a country ? If yes, explain why. If not, how will you modify it to give reliable results ? *8*

(b) What do you mean by stable and stationary population ? Derive the age distribution of a life table for stationary population. *8*

(c) Explain the system of collection and publication of Industrial Statistics in India. *8*

(d) Explain ratio to trend method for measurement of seasonal variation in a time series data. Discuss its merits and demerits. *8*

(e) If L(p) and P(q) represent Laspeyres' Index Number for prices and Paasche's Index Number for quantities respectively, show that

$$L(p) \times P(q) = V_{01}$$

where V_{01} is the value index number.

Hence or otherwise show that $\dfrac{L(p)}{L(q)} = \dfrac{P(p)}{P(q)}$

Also, show that if $A(p) = \dfrac{1}{2}[L(p) + P(q)]$, then A(p) is greater than the Fisher's Ideal Index Number. *8*

Q6. (a) For an autoregressive series of order two, namely

$$y_{t+2} + a\, y_{t+1} + b\, y_t = \varepsilon_{t+2}, \; |b| < 1$$

show that the correlogram ρ_k of order k is given by

$$\rho_k = p^k \frac{\sin(k\theta + \psi)}{\sin \psi}, \; p = +\sqrt{b}$$

where $\tan \psi = \dfrac{1+p^2}{1-p^2} \tan \theta, \quad \cos \theta = -\dfrac{a}{2p}$ *15*

(b) Explain an ARMA (p, q) process and ARIMA (p, d, q) process. How does Box-Jenkins (BJ) method facilitate the identification of ARMA or ARIMA model ? *10*

(c) Consider the general linear model given by

$$\mathbf{Y} = X\beta + \mathbf{U}$$

where $\mathbf{Y}$ is an $(n \times 1)$ vector observation on the explained variable

X is an $(n \times k)$ matrix of observations on the explanatory variable

$\mathbf{U}$ is an $(n \times 1)$ vector of error

β is $(k \times 1)$ vector of parameters to be estimated

Assume that $\mathbf{U} \sim N(0, \sigma^2 I_n)$ is independently and identically distributed and rank $(X) = k < n$.

Show that $\hat{\beta}$ is the least square estimator of β and is Best Linear Unbiased Estimator (BLUE) of β. Test the significance of the Null Hypothesis $H_0 : \beta = \beta_0$. *15*

Q7. (a) Define the force of mortality and show that its approximate value is

$$\mu_x = \frac{d_{x-1} + d_x}{2\,l_x}, \; x \geq 1.$$

If $l(x) = 100\sqrt{100 - x}$, find $\mu(84)$ exactly and by using approximate method. *15*

(b) State the scaling procedures developed for psychological research. Explain z-score and linear deviate score in brief.

The following table gives the mean and standard deviations of scores and the raw scores of two students in a number of examinations. Compare the overall performance of the two students. Give your conclusions. *15*

Examinations	Mean	Standard Deviation	Raw Score of Students	
			I	II
A	155·7	26·4	195	162
B	33·7	8·2	20	54
C	54·5	9·3	39	72
D	87·1	25·8	139	84
E	24·8	6·8	41	25

(c) What do you mean by projection of population ? Explain how you would do adjustment of migratory disturbances while projecting population. *10*

Q8. (a) Consider the following extended Keynesian model of income determination :

Consumption function : $C_t = \beta_1 + \beta_2 Y_t - \beta_3 T_t + U_{1t}$

Investment function : $I_t = \alpha_0 + \alpha_1 Y_{t-1} + U_{2t}$

Taxation function : $T_t = \gamma_0 + \gamma_1 Y_t + U_{3t}$

Income identity : $Y_t = C_t + I_t + G_t$

where C = Consumption expenditure

Y = Income

I = Investment

T = Taxes

G = Government expenditure

U's = the disturbance terms

In the model, the endogeneous variables are C, I, T and Y and the predetermined variables are G and Y_{t-1}.

By applying the order condition, check the identifiability of each of the equations in the system and of the system as a whole ? Write your conclusion. *15*

(b) Define Gross reproduction rate and Net reproduction rate. Distinguish between these two reproduction rates. Which one attains greater value ? Why ? Which one can be used for forecasting future population changes ? Why ? *15*

(c) Explain quasi-stable population. Stating the underlying assumptions, derive the age distribution function for quasi-stable population with changing mortality. *10*